JOURNAL OF COMPUTATIONAL PHYSICS 98, 178–180 (1992)

Abstracts of Papers to Appear in Future Issues

THE ARC TAN/TAN AND KEPLER–BURGER MAPPINGS FOR PERIODIC SOLUTIONS WITH A SHOCK, FRONT, OR INTERNAL BOUNDARY LAYER. John P. Boyd. *Department of Atmospheric, Oceanic, Space Science and Laboratory for Scientific Computation, 2200 Bonisteel Boulevard, Ann Arbor, Michigan 48109, USA.*

Many periodic solutions have internal regions of rapid change—internal boundary layers. Shock waves and geophysical fronts are one class of examples. A second class is composed of functions which decay rapidly away from a central peak or peaks. Spherical harmonics, Mathieu eigenfunctions, prolate spheroidal wave functions, and geophysical Hough functions may all be locally approximated by Hermite functions (in the appropriate parameter range) and decay exponentially fast outside a narrow subinterval. Similarly, the large amplitude cnoidal waves of the Korteweg–DeVries equation are narrow, isolated peaks which are well approximated by the $\operatorname{sech}^2(y)$ form of the solitary wave. In this article, we show that a change-of-coordinate is a powerful tool for resolving such internal boundary layers. In the first part, we develop a general theory of mappings for the spherical harmonic/cnoidal wave class of examples, which decay rapidly away towards the edges of the spatial period. The particular map $y = \arctan(L \tan(x))$ is a particularly effective choice. Four numerical examples show that this map and the Fourier pseudospectral method are a good team. In the second part, we generalize the earlier theory to describe mappings which asymptote to a constant but non-zero resolution at the ends of the periodicity interval. We explain why the "Kepler–Burger" mapping is particularly suitable for shock and fronts.

NUMERICAL COMPUTATION OF 2D SOMMERFELD INTEGRALS—DECOMPOSITION OF THE ANGULAR INTEGRAL. Steven L. Dvorak. *Electromagnetics Laboratory, Department of Electrical and Computer Engineering, University of Arizona, Tucson, Arizona 85721, USA*; Edward F. Kuester. *Electromagnetics Laboratory, Department of Electrical and Computer Engineering, Campus Box 425, University of Colorado, Boulder, Colorado 80309, USA.*

Spectral domain techniques are frequently used in conjunction with Galerkin's method to obtain the current distribution on planar structures. When this technique is employed, a large percentage of the computation time is spent filling the impedance matrix. Therefore, it is important to develop accurate and efficient numerical techniques for the computation of the impedance elements, which can be written as two-dimensional (2D) Sommerfeld integrals. Once the current distribution has been found, then the near-zone electric field distribution can be obtained by computing another set of 2D Sommerfeld integrals. The computational efficiency of the 2D Sommerfeld integrals can be improved in two ways. The first method, which is discussed in this paper, involves finding a new way to compute the inner angular integral in the polar representation of these integrals. It turns out that the angular integral can be decomposed into a finite number of incomplete Lipschitz–Hankel integrals, which in turn can be calculated using series expansions. Therefore, the angular integral can be computed by summing a series instead of applying a standard numerical integration algorithm. This new technique is found to be more accurate and efficient when piecewise-sinusoidal basis functions are used to analyze a printed strip dipole antenna in a layered medium. The incomplete Lipschitz–Hankel integral representation for the angular integral is then used in another paper to develop a novel asymptotic extraction technique for the outer semi-infinite integral.

NUMERICAL COMPUTATION OF 2D SOMMERFELD INTEGRALS—A NOVEL ASYMPTOTIC EXTRACTION TECHNIQUE. Steven L. Dvorak. *Electromagnetics Laboratory, Department of Electrical and Computer Engineering, University of Arizona, Tucson, Arizona 85721, USA*; Edward F. Kuester. *Electromagnetics Laboratory, Department of Electrical and Computer Engineering, Campus Box 425, University of Colorado, Boulder, Colorado 80309, USA.*

The accurate and efficient computation of the elements in the impedance matrix is a crucial step in the application of Galerkin's method to the analysis of planar structures. As was demonstrated in a previous paper, it is possible to decompose the angular integral, in the polar representation for the 2D Sommerfeld integrals, in terms of incomplete Lipschitz–Hankel integrals (ILHIs) when piecewise sinusoidal basis functions are employed. Since Bessel series expansions can be used to compute these ILHIs, a numerical integration of the inner angular integral is not required. This technique provides an efficient method for the computation of the inner angular integral; however, the outer semi-infinite integral still converges very slowly when a real axis integration is applied. Therefore, it is very difficult to compute the impedance elements accurately and efficiently. In this paper, it is shown that this problem can be overcome by using the ILHI representation for the angular integral to develop a novel asymptotic extraction technique for the outer semi-infinite integral. The usefulness of this asymptotic extraction technique is demonstrated by applying it to the analysis of a printed strip dipole antenna in a layered medium.

LOCATING THREE-DIMENSIONAL ROOTS BY A BISECTION METHOD. John M. Greene. *General Atomics, San Diego, California 92186-9784, USA.*

The evaluation of roots of equations is a problem of perennial interest. Bisection methods have advantages since the volume in which the root is known to be located can be steadily decreased. This method depends on the existence of a criterion for determining whether a root exists within a given volume. Here topological degree theory is exploited to provide this criterion. Only three-dimensional volumes are considered here. The result is of some use in locating roots and in illustrating the theory. The classification of roots as X-points or O-points and the generalization to three dimensions are also discussed.

CRYSTAL GROWTH AND DENDRITIC SOLIDIFICATION. James A. Sethian. *Department of Mathematics, University of California, Berkeley, California 94720, USA*; John Strain. *Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012, USA.*

We present a numerical method which computes the motion of complex solid/liquid boundaries in crystal growth. The model we solve includes physical effects such as crystalline anisotropy, surface tension, molecular

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